Beam, beam, beam

——空间光及SPP的波束解

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Outline

- Helmholtz Equation
- Gaussian beam
- Bessel beam
- Airy beam
- Mathieu and Weber beam
- Plasmonic counterpart

Diffraction
衍射

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Helmholtz Equation

- 电磁波波动方程

$$\nabla^2 \vec{B} - \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} = 0 \qquad \nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

- 代入时谐电磁场

$$\vec{E}(\vec{x},t) = \vec{E}(\vec{x})\exp(-i\omega t)$$
$$\vec{B}(\vec{x},t) = \vec{B}(\vec{x})\exp(-i\omega t)$$

- 可得 赫姆霍兹方程：

$$\nabla^2 u + k^2 u = 0$$

$$k^2 = (\frac{\omega}{c})^2$$

u代表E和B

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亥姆霍兹方程：$\nabla^2 \vec{E} + k^2 \vec{E} = 0$

最简单的解是：$\vec{E}(\vec{x}) = \vec{E}_0 e^{i\vec{k}\cdot\vec{x}}$ 平面电磁波

波形不随传播变化 → 无衍射?

wave不是beam

下面就来求解一个**beam**

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(I) Gaussian beam

亥姆霍兹方程的波束解

我们考虑的波束能量分布具有轴对称性，中间场强最大，靠近边缘强度迅速衰减。在横截面上具有这种分布性质的最简单函数就是高斯函数：

$$e^{-\frac{x^2+y^2}{w^2}}$$

参数w表示光束的宽度
波束宽度通常还是z的函数

波幅通常也是z的函数，我们以***u*(*x*,*y*,*z*)**代表电磁场任意直角分量，它可以具有如下形式

$$u(x,y,z)=g(z)e^{-f(z)(x^2+y^2)}e^{ikz}$$

$$u(x,y,z)=g(z)e^{-f(z)(x^2+y^2)}e^{ikz}$$

式中 e^{ikz} 表示以来与z的主要因子，剩下的因子中还有对z的缓变函数g(z)和f(z)，

因子 $e^{-f(z)(x^2+y^2)}$ 是限制波束的空间宽度的因子。

因子g(z)主要表示波的振幅，同时也含有传播因子中与纯平面因子 e^{ikz} 偏离的部分。令

$$\phi(x,y,z)=g(z)e^{-f(z)(x^2+y^2)}$$

它满足z的缓变振幅近似。因此它对z的展开式中的高次项可以忽略。

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根据亥姆霍兹方程 $\nabla^2 u + k^2 u = 0$

将 $u(x,y,z) = \phi(x,y,z)e^{ikz}$ 代入

得到：

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \left(\frac{\partial^2\phi}{\partial z^2} + 2ik\frac{\partial\phi}{\partial z} - k^2\phi\right) + k^2\phi = 0$$

忽略对z求导的高次项得到

$$\boxed{\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + 2ik\frac{\partial\phi}{\partial z} = 0}$$

傍轴波动方程

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将ϕ的试探解 $\phi(x,y,z)=g(z)e^{-f(z)(x^2+y^2)}$ 代入

$$(x^2+y^2)[2gf^2-ikgf']-[2fg-ikg']=0$$

上式要对任意x, y都成立，必须两括号内都为零

$$\begin{cases} 2f^2=ikf' & (1) \\ 2fg=ikg' & (2) \end{cases}$$

上式（1）的解形式为

$$f(z)=\frac{1}{A+\frac{2i}{k}z}$$

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对比（1）（2）式，可以看出**g(z)=f(z)**×常数

$$g(z)=\frac{u_0}{1+\frac{2i}{kA}z}$$

将**f(z)**变换，得到

$$f(z)=\frac{1}{A(1+\frac{4z^2}{k^2A^2})}(1-\frac{2i}{kA}z)$$

令 $A=w_0^2$

$$w^2(z)=A(1+\frac{4z^2}{k^2A^2})=w_0^2[1+(\frac{2z}{kw_0^2})^2]$$

则：

$$f(z)=\frac{1}{w^2(z)}(1-\frac{2iz}{kw_0^2})$$

那么高斯函数变为

$$e^{-f(z)(x^2+y^2)}=\exp\left[-\frac{x^2+y^2}{w^2(z)}(1-\frac{2iz}{kw_0^2})\right]$$

同样**g(z)** 可以写为

$$g(z)=\frac{u_0}{\sqrt{1+(\frac{2z}{kw_0^2})^2}}e^{-i\varphi}=u_0\frac{w_0}{w}e^{-i\varphi}$$

其中 $\varphi=\arctan(\frac{2z}{kw_0^2})$

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最终得到光束场强函数

$$u(x,y,z)=u_0\frac{w_0}{w}e^{-\frac{x^2+y^2}{w^2}}e^{-i\Phi}$$

其中

$$\Phi=kz+\frac{k(x^2+y^2)}{2z\left[1+(\frac{kw_0^2}{2z})^2\right]}-\varphi$$

$$w^2(z)=w_0^2[1+(\frac{2z}{kw_0^2})^2]$$

高斯光束的传播特性

$$u(x,y,z)=u_0\frac{w_0}{w}e^{-\frac{x^2+y^2}{w^2}}e^{-i\Phi}$$

振幅　　相因子

$e^{-\frac{x^2+y^2}{w^2}}$ 限制波束宽度

$$w^2(z)=w_0^2[1+(\frac{2z}{kw_0^2})^2]$$

波束宽度由w(z)代表，在z=0处波束具有最小宽度，称为光束腰部——束腰。

$u_0\frac{w_0}{w}$　z轴上的振幅，u_0是束腰位置的振幅。

$$\Phi = kz + \frac{k(x^2+y^2)}{2z\left[1+\left(\frac{kw_0^2}{2z}\right)^2\right]} - \varphi$$

光束波阵面是等相位面，由相位Φ=常数确定。

当z=0时，Φ=0，说明z=0平面是一个波阵面，即光束腰部波阵面与z轴垂直。

当远离束腰部位　$z >> kw_0^2$

$$z + \frac{x^2+y^2}{2z} = 常数$$

再由 $z^2 >> x^2 + y^2$

$$(1+\frac{x^2+y^2}{z^2})^{\frac{1}{2}} \approx 1+\frac{x^2+y^2}{2z^2}$$

所以等相位面方程可写为:

$$z(1+\frac{x^2+y^2}{z^2})^{\frac{1}{2}} \approx 常数$$

即 $r=(x^2+y^2+z^2)^{\frac{1}{2}} \approx 常数$

因此，在远处波阵面变为以束腰中心为球心的一个球面。总的波阵面就是从腰部的平面逐渐过渡到远处的球面形状。

另外，在远处 $z >> kw_0^2$

$$w(z) = w_0^2[1+(\frac{2z}{kw_0^2})^2] \approx \frac{2z}{kw_0^2}$$

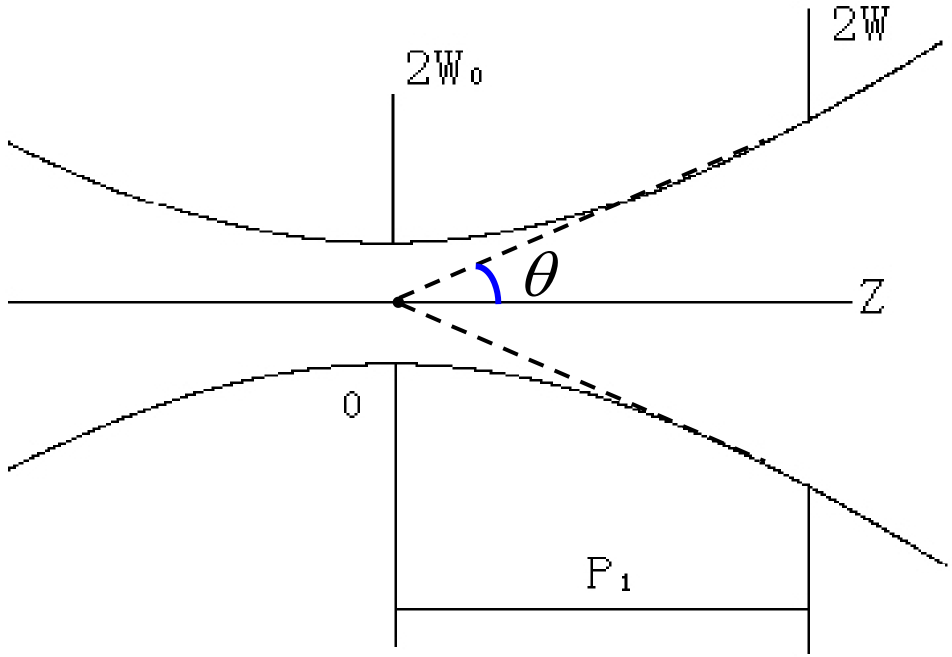
波束发散角由tanθ=w/z确定，所以

$$\tan\theta \approx \frac{2}{kw_0^2}$$

对应一给定波长的电磁波，当w_0愈小时，发散角愈大。因此，要求良好的聚焦效果，发散角必须足够大；如果要求良好定向，则光束宽度不能太小。

$$\Delta k_\perp \cdot w_0 \approx O(1)$$

高斯光束示意图



光束在传播中有发散，明显的衍射效应

Q1：是否存在不衍射的光束？

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(II) Bessel beam

Helmhotz Equation $\nabla^2 u + k^2 u = 0$

在柱坐标下变为

$$\frac{1}{r}\frac{\partial u}{\partial r}(\frac{1}{r}\frac{\partial u}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} + k^2 u = 0$$

令$u(r,\theta,z) = v(r,\theta)Z(z)$,代入方程：

$$Z[\frac{1}{r}\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial v}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} + k^2 v] = -v\frac{d^2 Z}{dz^2}$$

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作分离变量，引入常数λ，得到：

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial v}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}+(k^2-\lambda)v=0$$

$$\frac{d^2 Z}{dz^2}+\lambda Z=0$$

接着令$v(r,\theta)=R(r)\Phi(\theta)$,代入得到：

$$\frac{r^2}{R}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+(k^2-\lambda)R\right]=-\frac{1}{\Phi}\frac{d^2\Phi}{d\theta^2}$$

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最终得到三个分离变量微分方程：

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+\left(k^2-\lambda-\frac{\mu}{r^2}\right)R=0$$

$$\frac{d^2\Phi}{d\theta^2}+\mu\Phi=0 \qquad \Phi_m(\theta)=\begin{cases}\cos(m\theta)\\ \sin(m\theta)\end{cases} \text{或} \begin{cases}\exp(im\theta)\\ \exp(-im\theta)\end{cases}$$

$$\frac{d^2Z}{dz^2}+\lambda Z=0 \qquad Z_\lambda(z)=\begin{cases}\cos(k_z z)\\ \sin(k_z z)\end{cases} \text{或} \begin{cases}\exp(ik_z z)\\ \exp(-ik_z z)\end{cases}$$

这里$\mu=m^2,\lambda=k_z^2$。

令$k_r^2=k^2-k_z^2=k^2-\lambda$。此时方程（1）变为

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+\left(k_r^2-\frac{m^2}{r^2}\right)R=0$$

Bessel 方程

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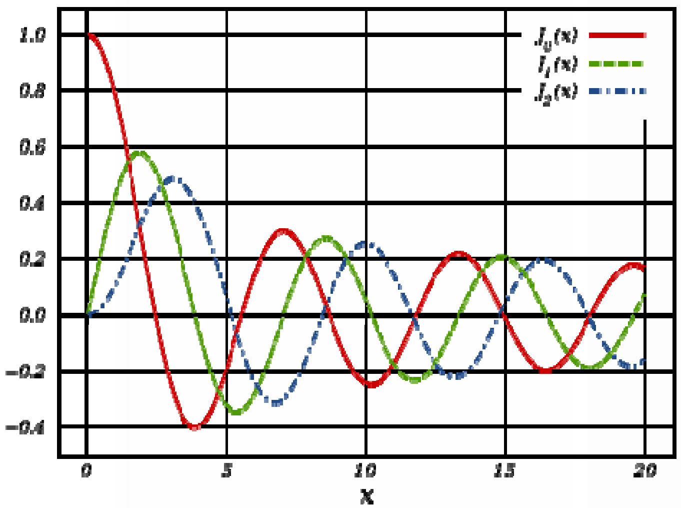
因此我们得到径向函数的通解

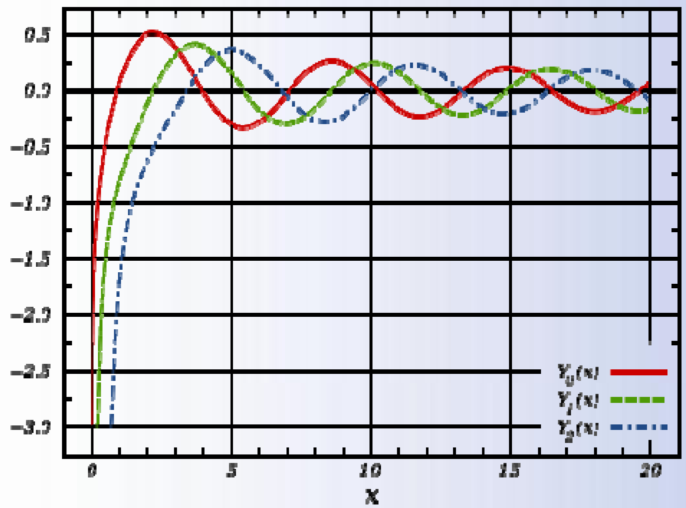
$$R(r) = CJ_m(k_r r) + DN_m(k_r r)$$

由边界R(0)有限值，得到D=0。

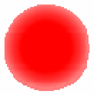
所以最终的径向解就是Bessel函数**J_m(kr)**形式，

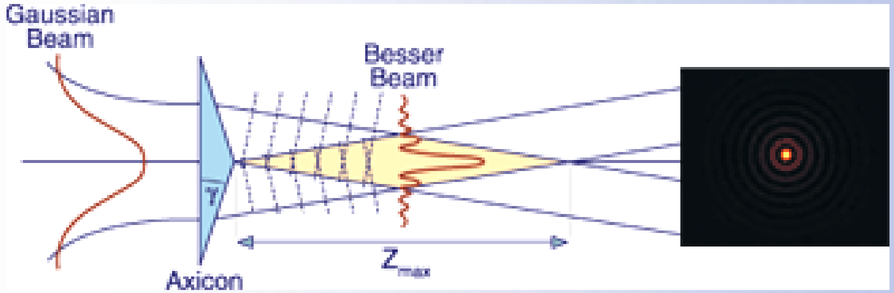
而旋转切向则为振荡解，振荡波矢**m**决定J的阶数！

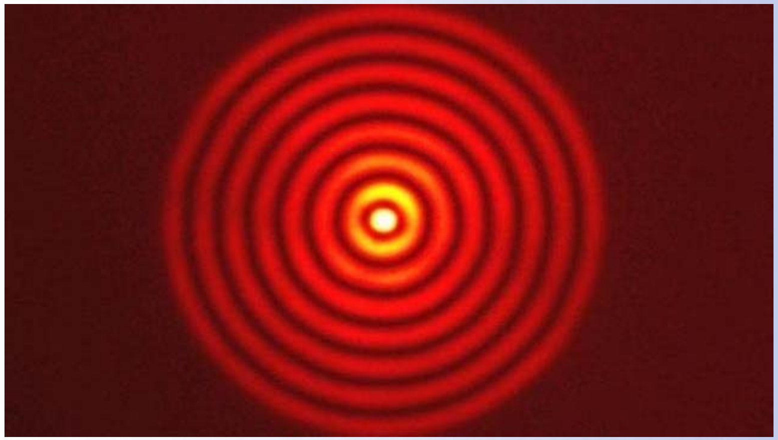


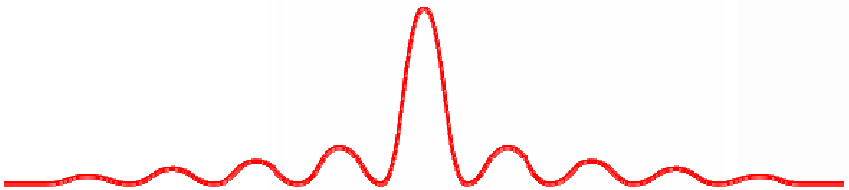


零阶Bessel beam （m=0）









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Diffraction-Free Beams

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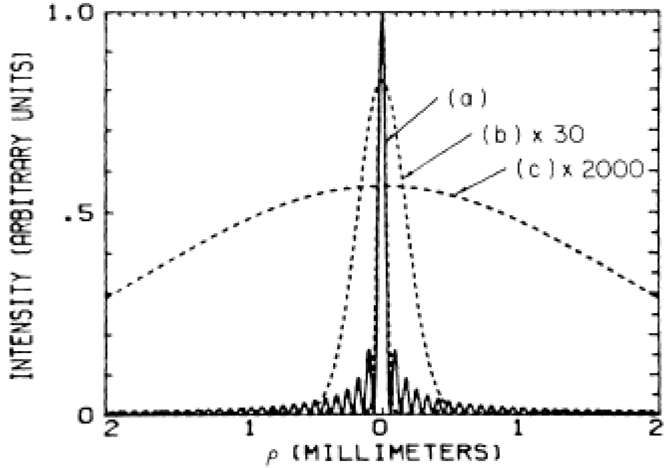
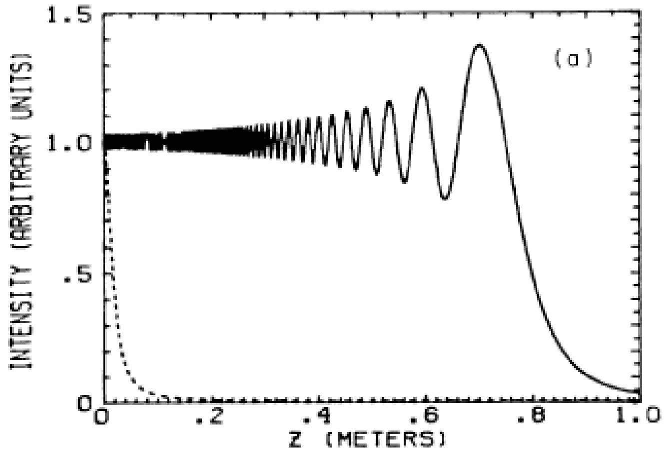
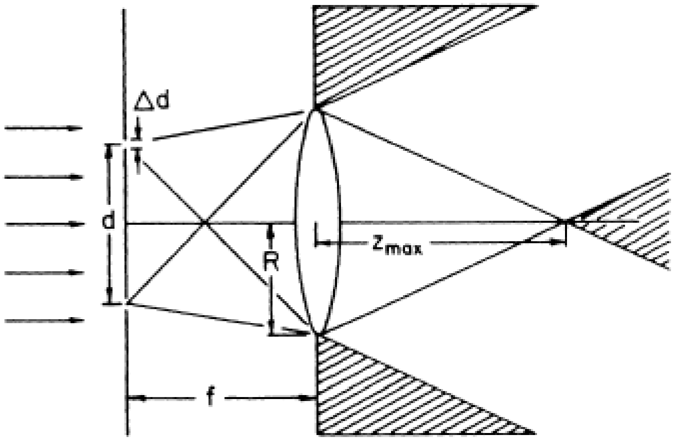
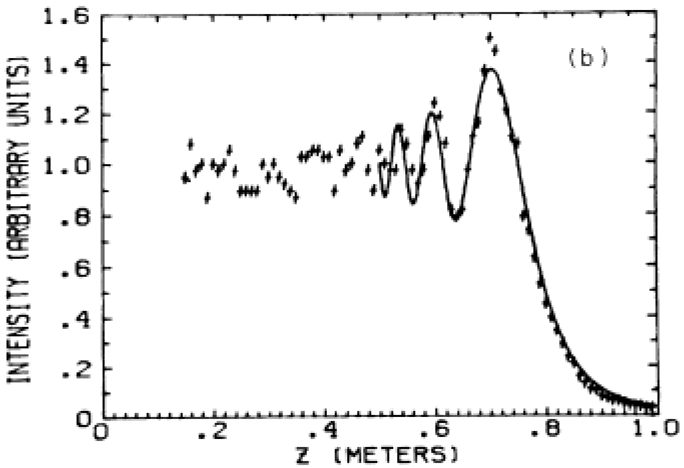
The Institute of Optics, University of Rochester, Rochester, New York 14627









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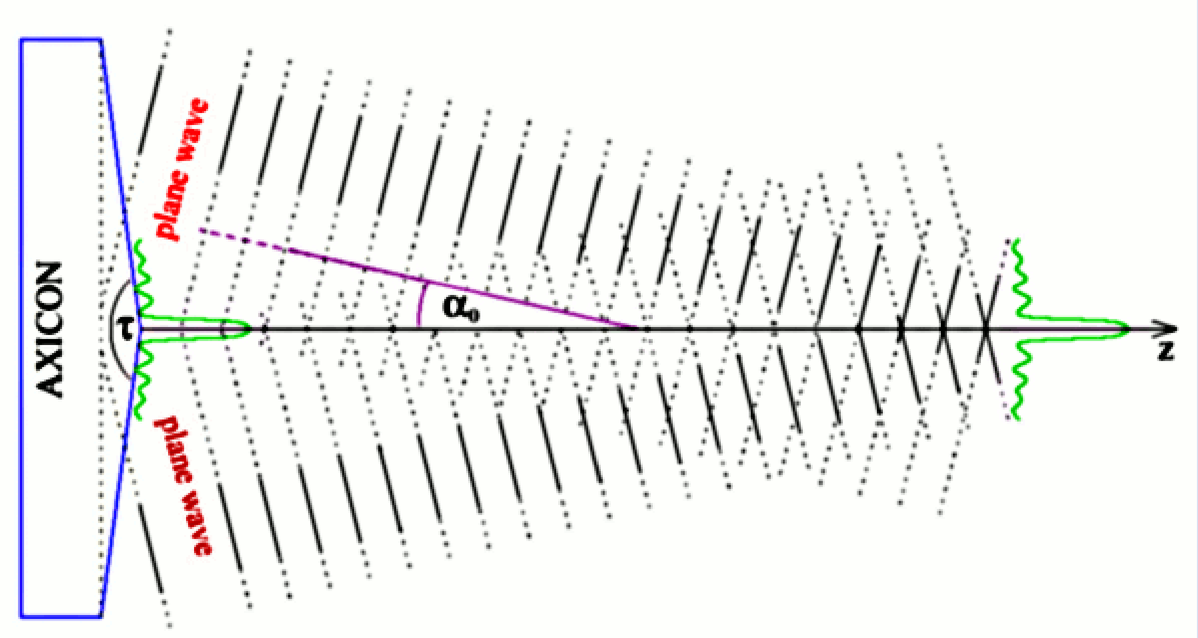

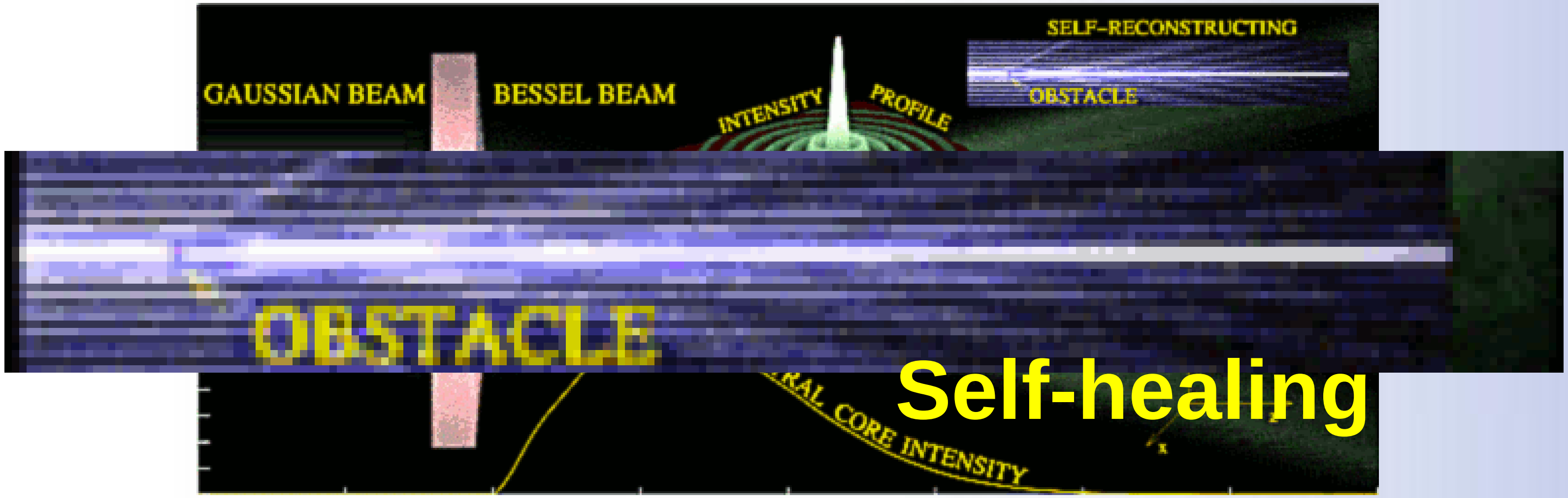

Self-healing

http://www.isibrno.cz/omitec/index.php?action=bessintr.html

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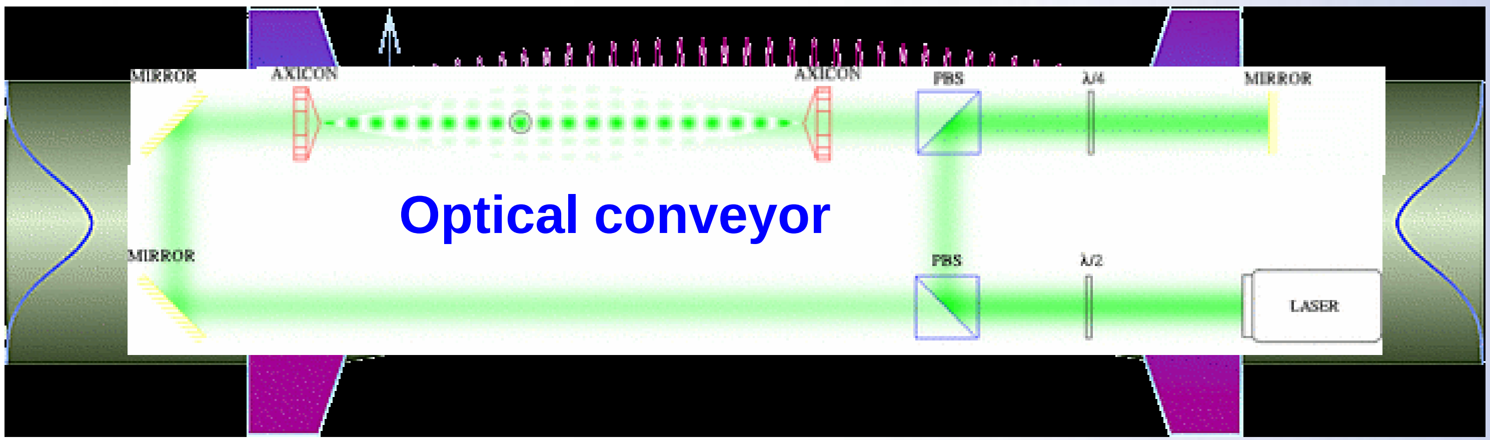
Q2：是否存在弯曲的 nondiffracting beam?



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(III) Airy beam

- 回到Cartesian傍轴波动方程

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+2ik\frac{\partial\phi}{\partial z}=0$$

- 如果考虑场在y方向均匀，即无波动，则方程变为二维傍轴方程，如下

$$\frac{\partial^2\phi}{\partial x^2}+2ik\frac{\partial\phi}{\partial z}=0$$

形式一致！

- 对照薛定谔方程

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}=i\hbar\frac{\partial\psi}{\partial t}$$

如何求解？

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Airy 函数

*Airy*方程：$y''-xy=0$

$y=\mathrm{Ai}(x)$是方程的一支解。

$$\mathrm{Ai}(x)=\frac{1}{\pi}\int_0^{\infty}\cos\left(\frac{t^3}{3}+xt\right)dt$$

还有一支

$$\mathrm{Bi}(x)=\frac{1}{\pi}\int_0^{\infty}\left[\exp\left(-\frac{t^3}{3}+xt\right)+\sin\left(\frac{t^3}{3}+xt\right)\right]dt$$

真正的通解：

$$y=C_1\,\mathrm{Ai}(x)+C_2\,\mathrm{Bi}(x)$$

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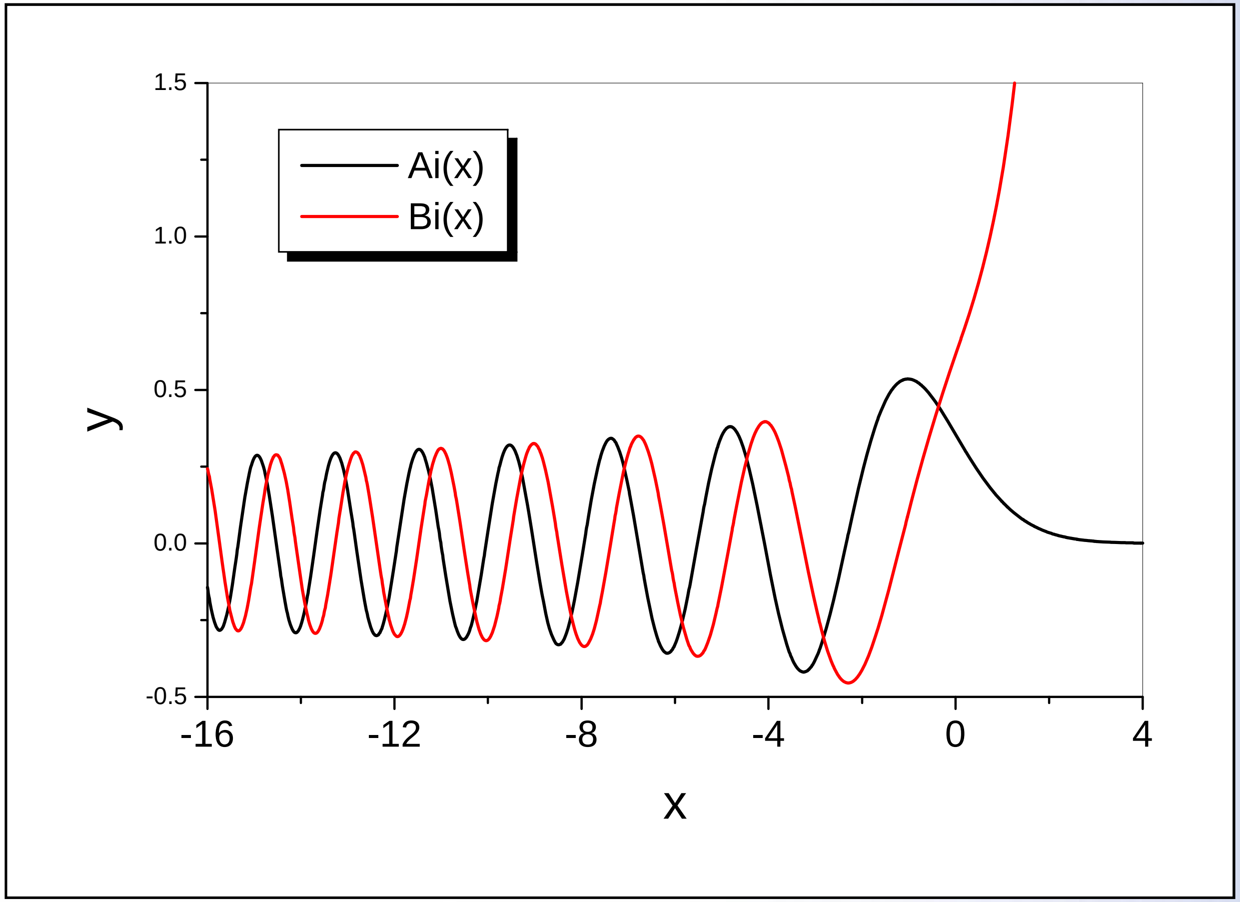
Ai(x) Bi(x) 函数图象



Ai(x)函数具体形式

$$\text{Airy方程：}\frac{d^2 Ai(x)}{dx^2} - xAi(x) = 0$$

对其做变量变换 $\xi = x^{3/2}, w(\xi) = Ai(x)/\sqrt{x}$

得到：

$$\frac{d^2 w}{d\xi^2} + \frac{1}{\xi}\frac{dw}{d\xi} - (\frac{1}{3})^2(4 + \frac{1}{\xi^2})w = 0$$

虚宗量贝塞尔方程

$$\mathrm{Ai}(x) = \begin{cases} \frac{1}{\pi}\sqrt{\frac{x}{3}}\mathrm{K}_{1/3}\left(\frac{2}{3}x^{\frac{3}{2}}\right), & (x>0) \\ \frac{\sqrt{-x}}{3}\left[\mathrm{J}_{1/3}\left(\frac{2}{3}(-x)^{\frac{3}{2}}\right) + \mathrm{J}_{-1/3}\left(\frac{2}{3}(-x)^{\frac{3}{2}}\right)\right], & (x<0) \end{cases}$$

贝塞尔函数的马甲

如何根据Ai(x)求薛定谔方程： $\frac{\partial^2 \phi}{\partial x^2} + 2ik\frac{\partial \phi}{\partial z} = 0$

不失一般性令$\phi = \mathrm{Ai}(f_1)\exp(f_2)$，其中$f_1$和$f_2$是$x, z$的函数。

然后求出$\frac{\partial \phi}{\partial z}$、$\frac{\partial^2 \phi}{\partial x^2}$的具体形式，代入波动方程，并利用：

$$\mathrm{Ai}''(x) = x\,\mathrm{Ai}(x)$$

最后对比$\mathrm{Ai}(x)$和$\mathrm{Ai}'(x)$前面的系数，可得：

$$\begin{cases} \frac{\partial^2 f_1}{\partial x^2} + 2\frac{\partial f_1}{\partial x}\frac{\partial f_2}{\partial x} = -2ik\frac{\partial f_1}{\partial z} \\ \left(\frac{\partial f_2}{\partial x}\right)^2 + \frac{\partial^2 f_2}{\partial x^2} + f_1\left(\frac{\partial f_1}{\partial x}\right)^2 = -2ik\frac{\partial f_2}{\partial z} \end{cases}$$

考虑无衍射形式的解，这要求$f_1 = ax + f_3(z)$，其中f_3仅是z的函数。代入上述方程组，然后分离变量，并考虑初值条件，最后可解得：

$$f_3(z) = -\frac{a^4}{4k^2} z^2,$$

$$f_1(x,z) = ax - \frac{a^4 z^2}{4k^2},$$

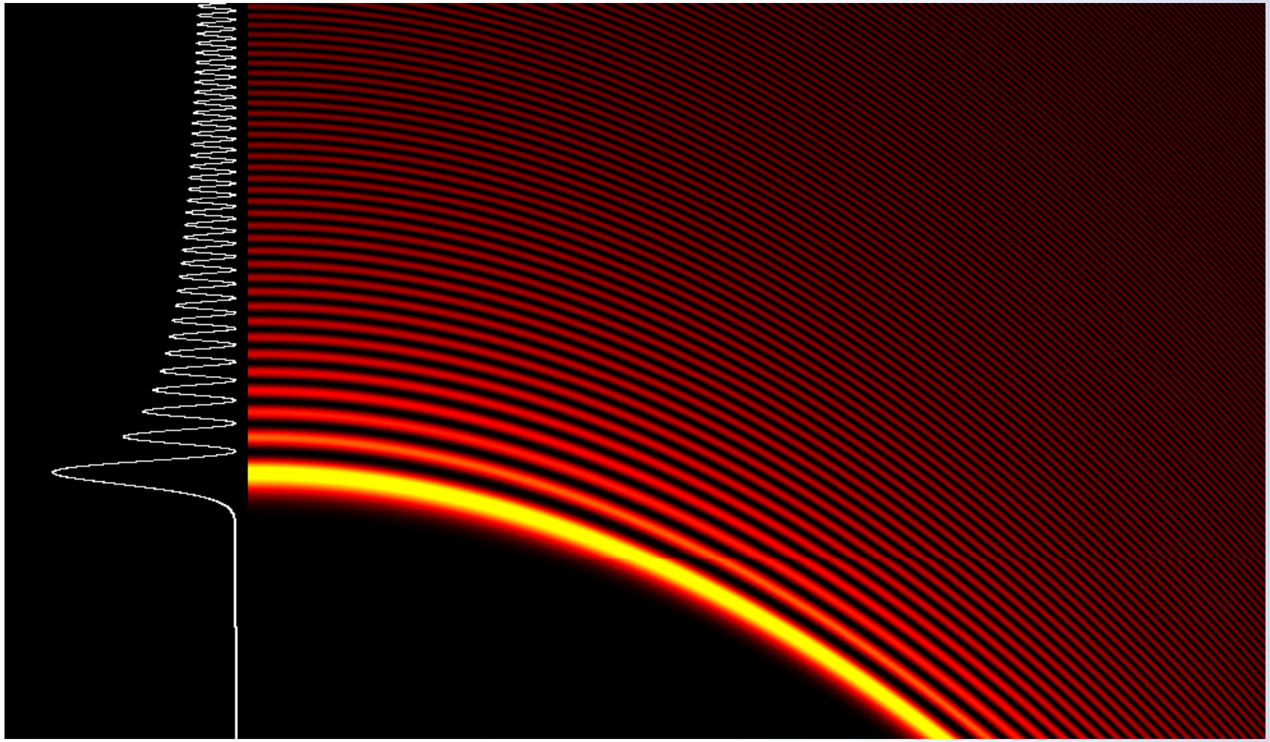
$$f_2(x,z) = i\frac{a^6 z^3}{12k^3} - i\frac{a^3 xz}{2k}.$$

将f_1和f_2代入ϕ的表达式即可得到最后结果。

$\phi(x,0) = \mathrm{Ai}(ax)$初值条件下得到非衍射严格解：

$$\phi(x,z) = \mathrm{Ai}\left(ax - \frac{a^4 z^2}{4k^2}\right)\exp\left(i\frac{a^6 z^3}{12k^3} - i\frac{a^3 xz}{2k}\right)$$

Airy beam图象



不衍射，不发散，
自弯曲，自加速，自修复

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Nonspreading wave packets

$$\psi(x,0) \approx \frac{1}{\sqrt{\pi}}\left(\frac{\hbar^{2/3}}{-Bx}\right)^{1/4} \sin[\pi/4 + 2(-Bx)^{3/2}/3\hbar]$$

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Observation of Accelerating Airy Be

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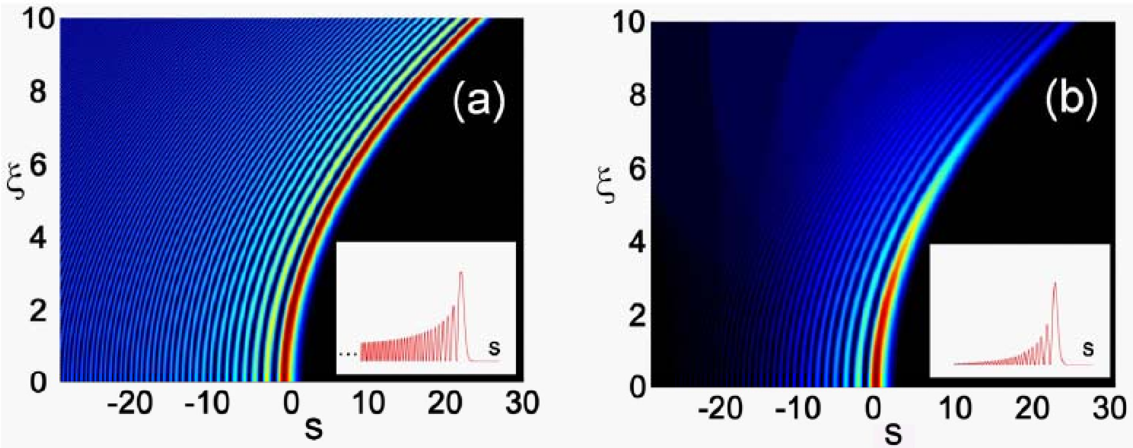
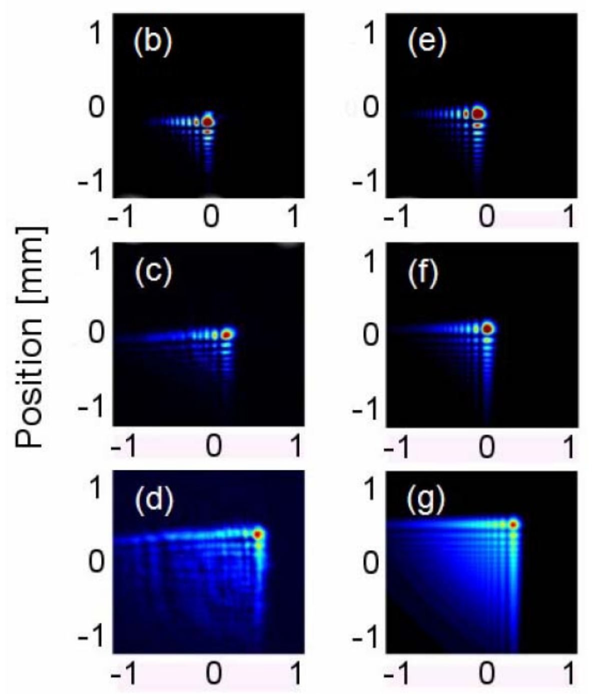
(Received 15 August 2007; published 20 Novembe





Infinite → finite

$$i\frac{\partial \phi}{\partial \xi} + \frac{1}{2}\frac{\partial^2 \phi}{\partial s^2} = 0,$$

$$\phi(\xi, s) = \mathrm{Ai}(s - (\xi/2)^2)\exp(i(s\xi/2) - i(\xi^3/12)).$$

有限能量 $\phi(0, s) = \mathrm{Ai}(s)\exp(as)$

$$\phi(\xi, s) = \mathrm{Ai}(s - (\xi/2)^2 + ia\xi)\exp(as - (a\xi^2/2) - i(\xi^3/12) + i(a^2\xi/2) + i(s\xi/2)).$$

Fourier Transformation

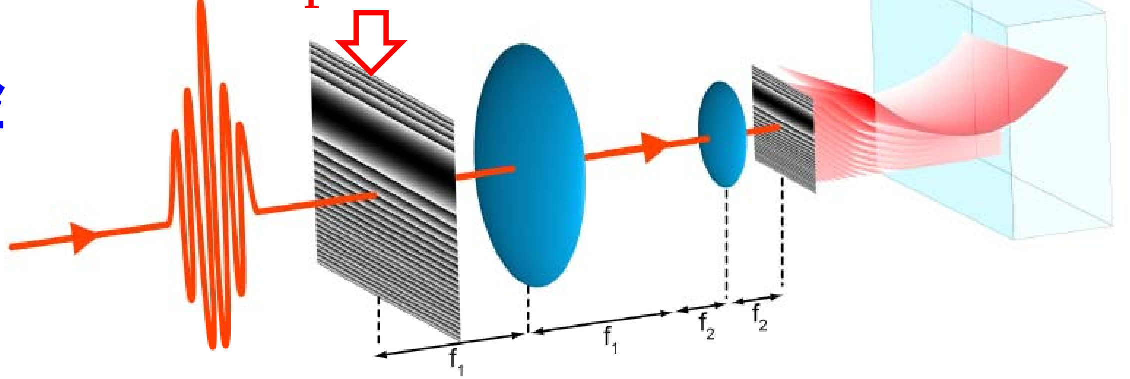
$$\Phi(k) \propto \exp(-ak^2)\exp(ik^3/3)$$

高斯光束 ← $\exp(-ak^2)$

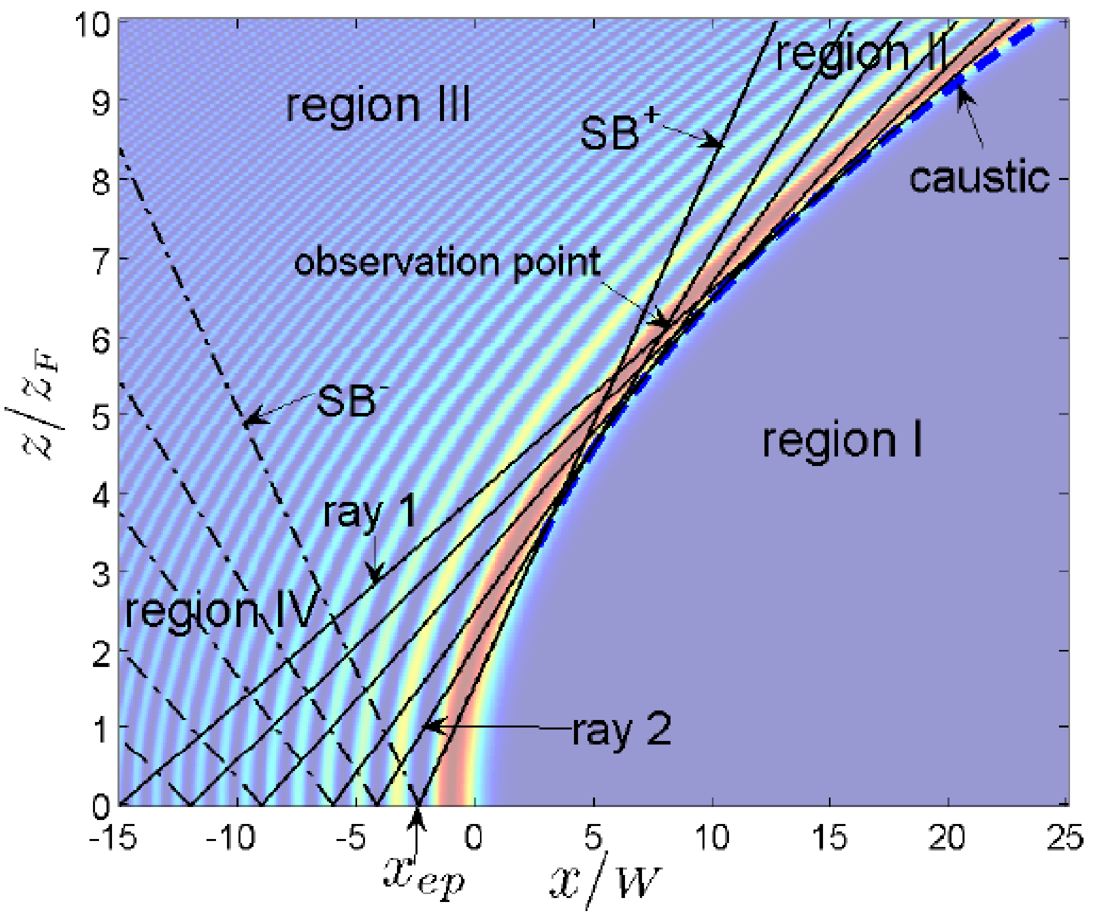
附加相位调制 — $\exp(ik^3/3)$

Cubic phase mask

实验

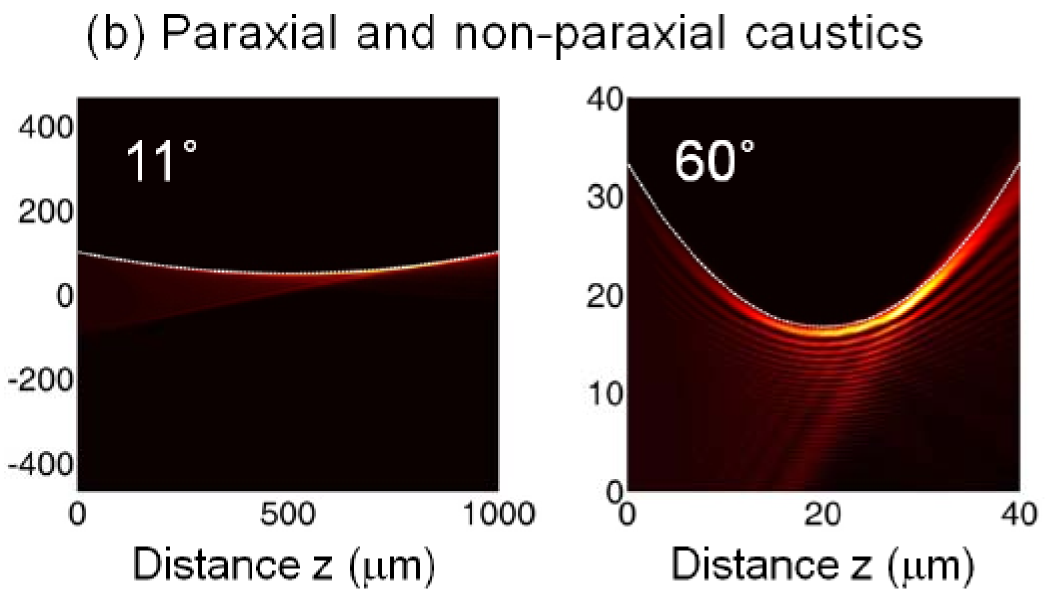


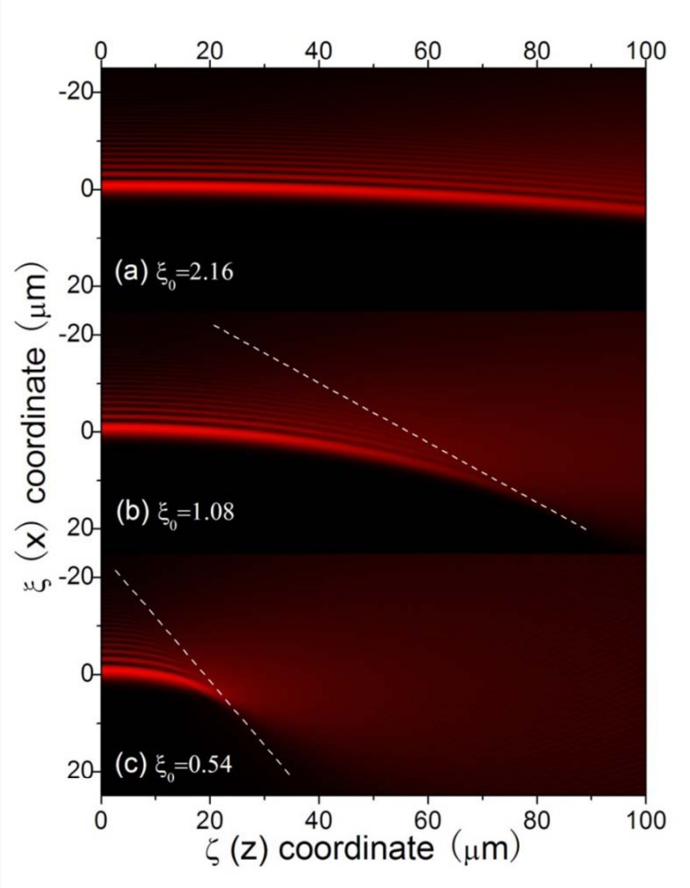
Geometric optics illustration



傍轴问题（paraxial）

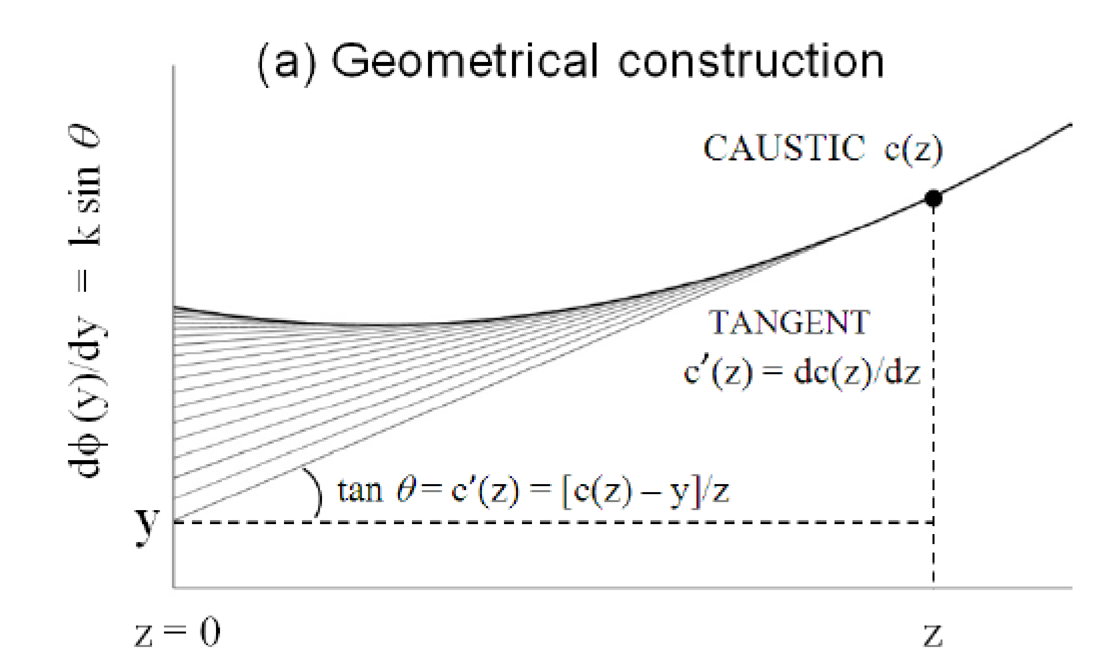
$$\psi(x) = -\frac{2}{3}\left(-\frac{\xi}{\xi_0}\right)^{3/2} - \frac{\pi}{4} - k\frac{\xi\sin\theta}{\cos\left(\theta-\theta_\xi\right)}$$





?

Arbitrary caustic beam



$$\frac{d\phi(y)}{dy} = k \sin\theta = \frac{k\, c'(z)}{\sqrt{1+[c'(z)]^2}}$$

Paraxial approximation: tanθ≈sinθ≈θ

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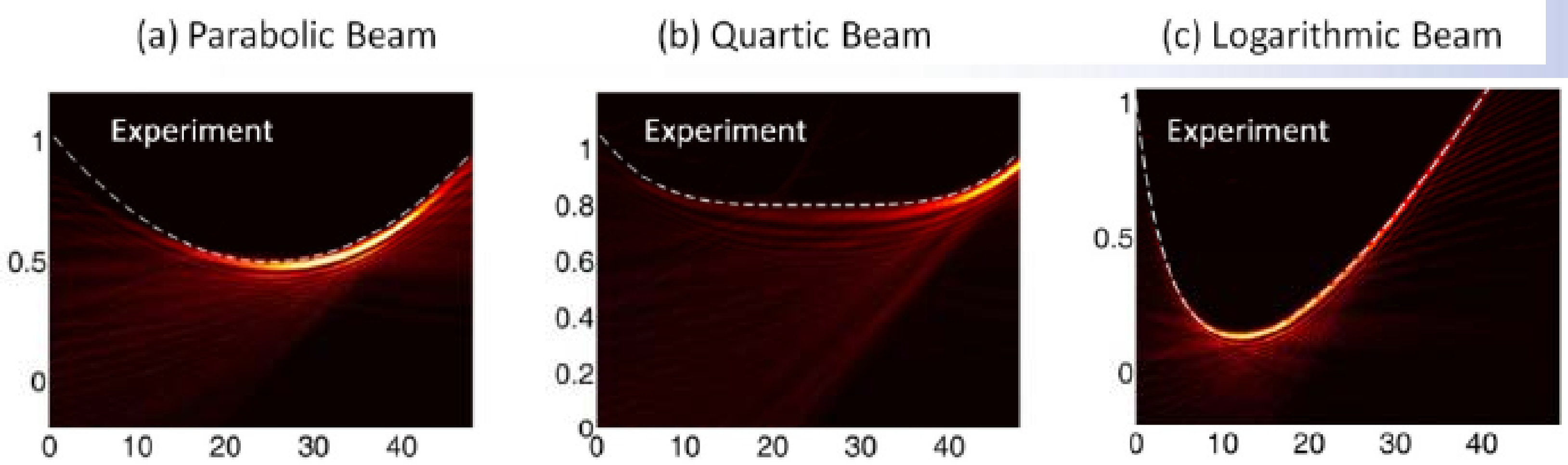
Airy beam

Table 1. In the Paraxial Approximation, this Table gives the Calculated Phase for Desired Acceleration Profiles as Shown.

Acceleration profile	Applied phase
Parabolic: $c(z) = az^2$	$\phi(y) = -4/3\, a^{1/2}\, k\, y^{3/2}$
Quartic: $c(z) = az^4$	$\phi(y) = -16/21\, (3a)^{1/4}\, k\, y^{7/4}$
Logarithmic: $c(z) = a \ln(bz)$	$\phi(y) = e^{-1} a^2 b\, k\, (1-\exp[-y/a])$
Polynomial: $c(z) = az^n$ (for n even)	$\phi(y) = kn^2 y^2 \dfrac{[a(1-n)/y]^{1/n}}{(2n-1)(1-n)}$



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Nonparaxial $\overset{?}{=}$ nondiffracting

数学上：Airy 函数解是Helmhotz方程傍轴近似下的严格解。

实验上：当caustic beam角度变大后，beam明显偏离原来的profile。

Q3：是否存在 nonparaxial nondiffracting beam?

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Outline

- Helmholtz Equation
- Gaussian beam
- Bessel beam
- Airy beam
- **Mathieu and Weber beam**
- Plasmonic counterpart

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角向Bessel光束

- 回到最原始波动方程

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}+\frac{1}{c^2}\frac{\partial\phi}{\partial t}=0$$

- 考虑场在z方向均匀，则变为二维Helmhotz方程

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{1}{c^2}\frac{\partial\phi}{\partial t}=0$$

- 分离变量令 $\phi=U(r)\exp(i\alpha\theta-i\omega t)$
  则径向函数满足

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial U}{\partial r})+(k^2-\frac{\alpha^2}{r^2})U=0$$

标准的Bessel方程

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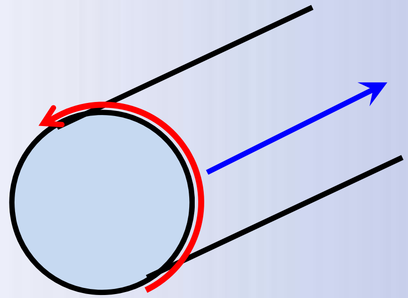
• 如此得到径向不衍射解



$$U(r) = J_\alpha(kr)$$

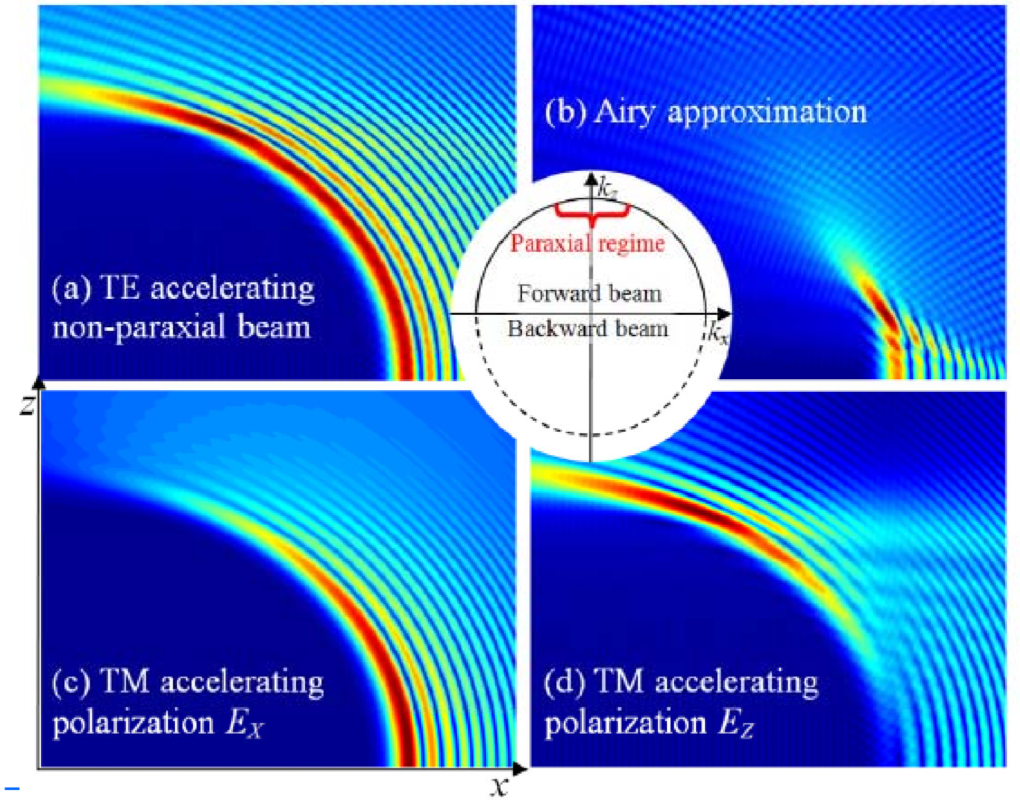
$$\phi = J_\alpha(kr)\exp(i\alpha\theta - i\omega t)$$

注意：α表示方位角向θ的行波波数，在求Bessel光束时候我们

但是波沿着z轴传播$k_z \neq 0$。

\而 α≠0，得高阶Bessel解，

寸弯曲波。



这里α=100（高阶贝塞尔函数）
这时beam弯曲接近90度，
大大突破了paraxial的限制！

刚才的结果是在柱坐标下完成，其实可以进一步推广到椭圆坐标、抛物坐标等任意曲线坐标系下：

椭圆坐标系

$$\begin{cases} x = h\sinh\xi\sin\eta \\ y = h\cosh\xi\cos\eta \end{cases}$$

$\xi\in[0,\infty)$对应于“径向”
$\eta\in[0,2\pi)$对应于“方位角”
$h=\sqrt{|a^2-b^2|}$

$$\frac{d^2R(\xi)}{d\xi^2}-(\beta-2q\cosh 2\xi)R(\xi)=0$$

$$\frac{d^2\Theta(\eta)}{d\xi^2}-(\beta-2q\cosh 2\xi)\Theta(\eta)=0$$

$$M(\xi,q)=R_m(\xi,q)(ce_m(\eta;q)-ise_m(\eta;q))$$

径向Mathieu函数（$R_m(\xi,q)$）

角向Mathieu函数（$ce_m(\eta;q)-ise_m(\eta;q)$）

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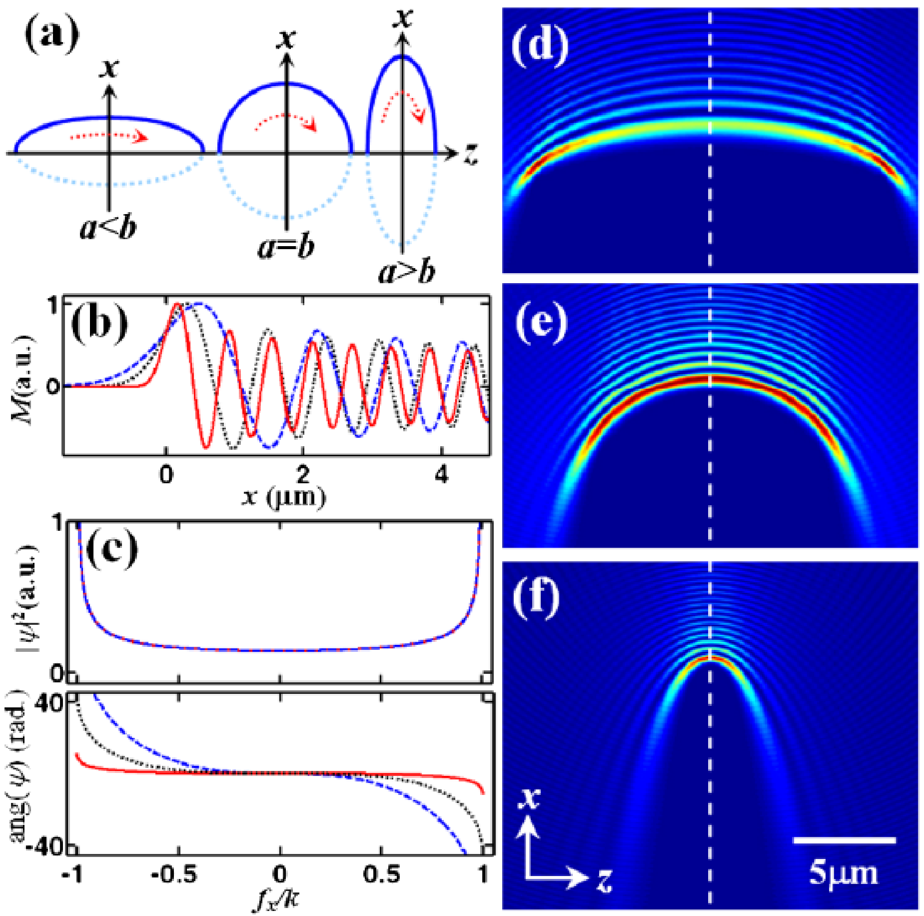
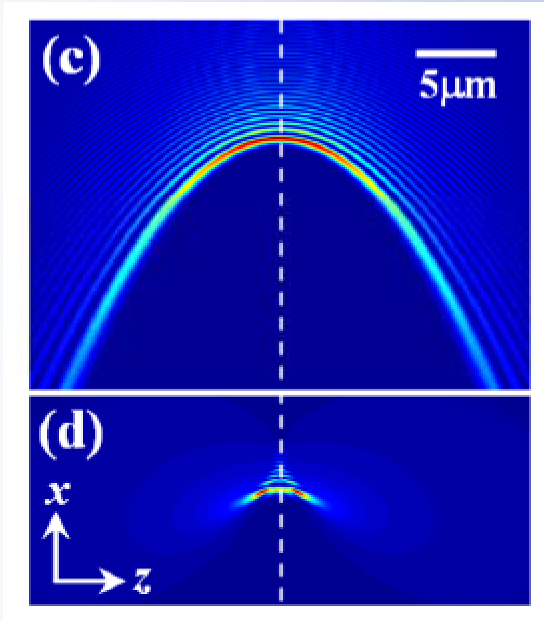
Mathieu beam



Parabolic coordinate

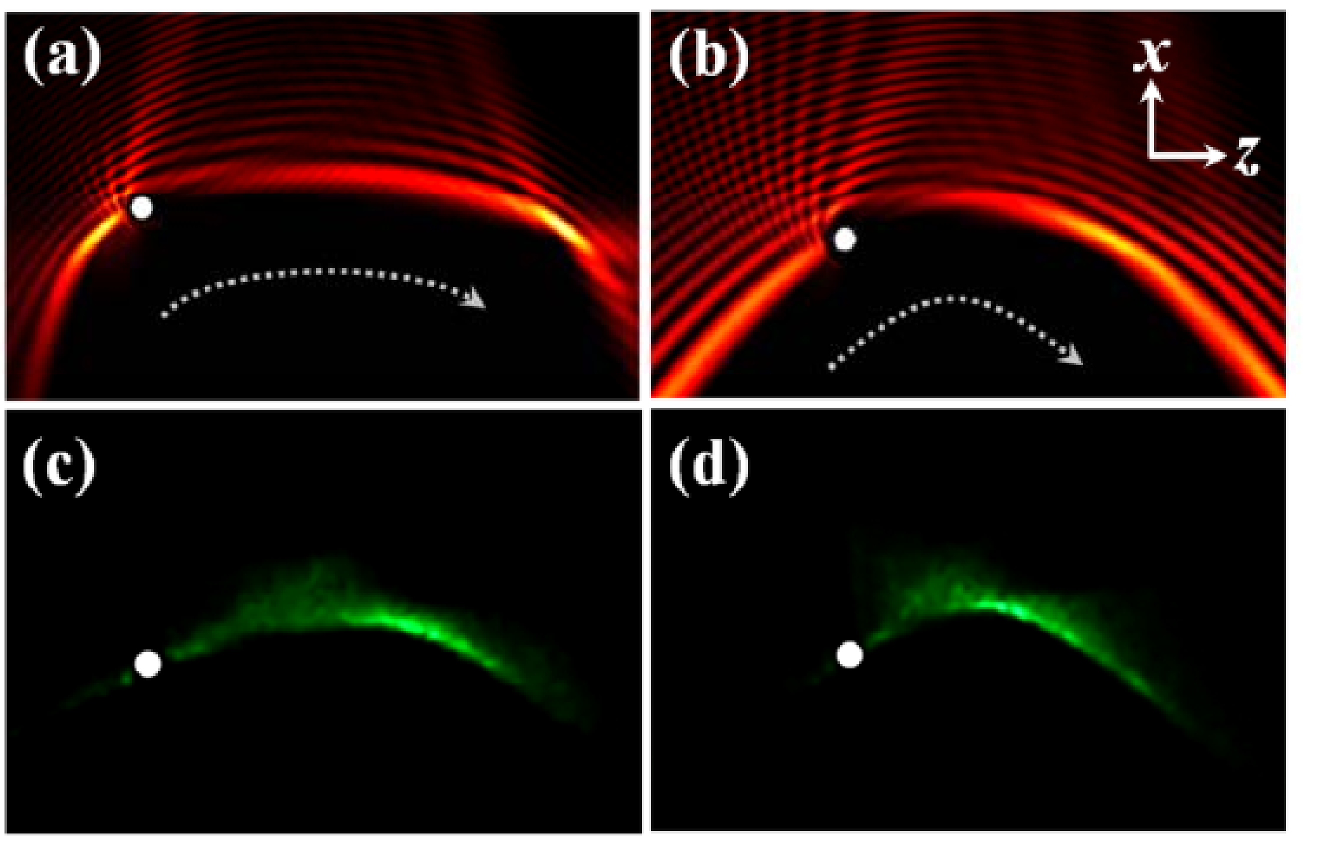
$$\frac{d^2\Phi(\sigma)}{d\sigma^2} + (k^2\sigma^2 + 2k\gamma)\Phi(\sigma) = 0,$$

$$\frac{d^2\vartheta(\tau)}{d\tau^2} + (k^2\tau^2 - 2k\gamma)\vartheta(\tau) = 0,$$



Weber beam

Self-healing property



Outline

- Helmholtz Equation
- Gaussian beam
- Bessel beam
- Airy beam
- Mathieu and Weber beam
- **Plasmonic counterpart**

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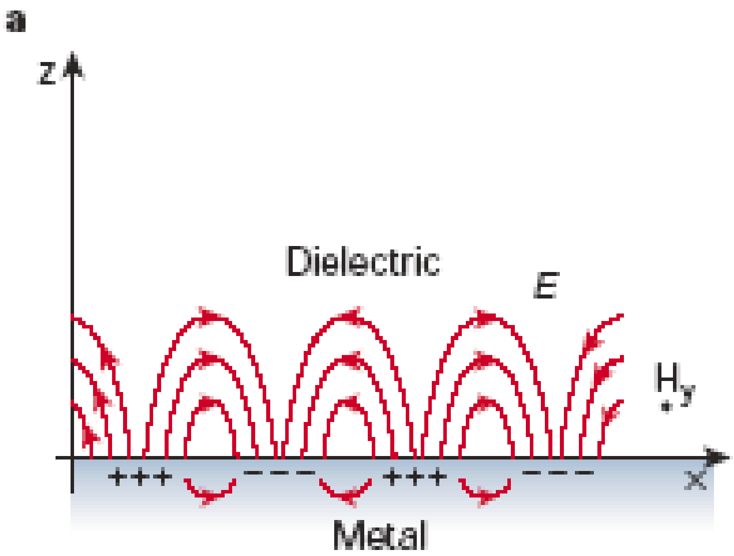
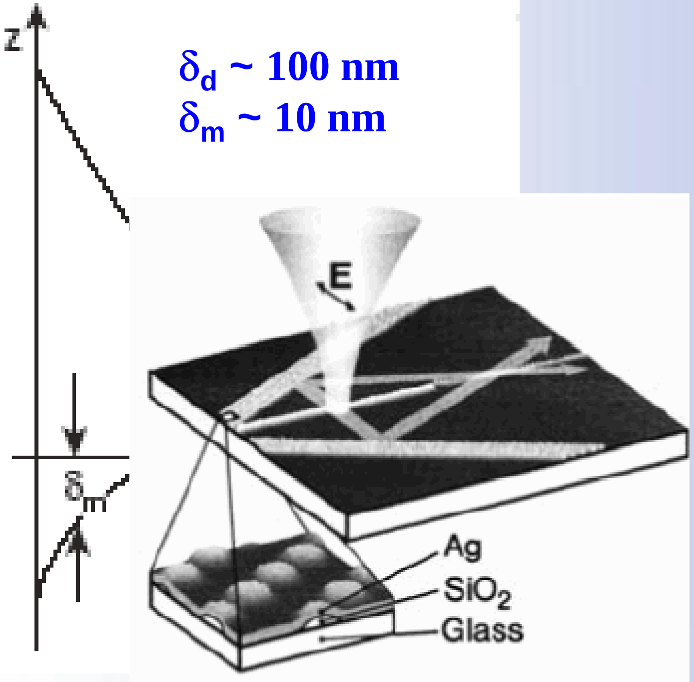
(IV) Plasmon counterparts

Two dimensional nature of SPP



δ_d ~ 100 nm
δ_m ~ 10 nm



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Airy plasmon: a nondiffracting surface wave

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Received March 1, 2010; accepted May 17, 2010;
posted June 2, 2010 (Doc. ID 124913); published June 11, 2010

针对SPP的TM场的方程

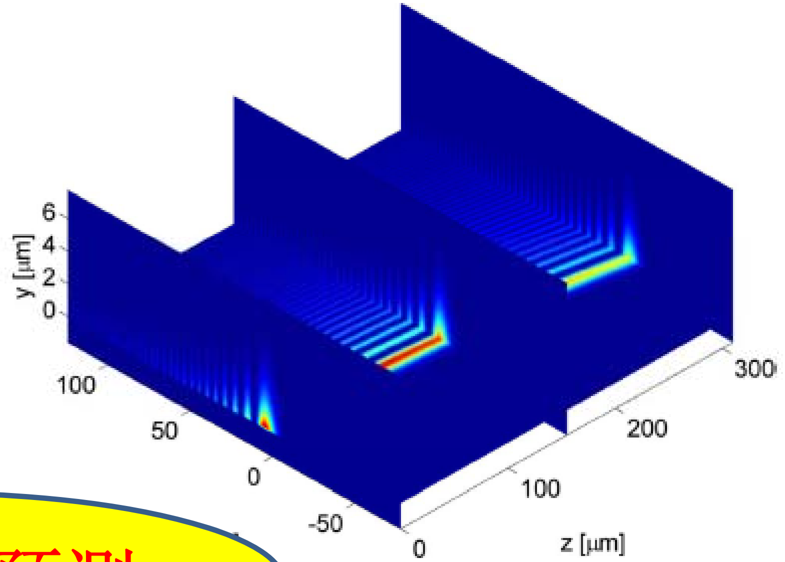
$$\nabla^2 E_{dy} + k_0^2 \varepsilon_d E_{dy} = 0.$$

$$\frac{\partial^2 A}{\partial x^2} + 2ik_z \frac{\partial A}{\partial z} = 0.$$

$k_z > k_0$
亚波长特性

很容易也可得到Airy函数的解

$$A(x,z) = Ai\left[\frac{x}{x_0} - \left(\frac{z}{2k_z x_0^2}\right)^2 + i\frac{az}{k_z x_0^2}\right] \times \exp\left[i\left(\frac{x + a^2 x_0}{2x_0}\frac{z}{k_z x_0^2} - \frac{1}{12}\left(\frac{z}{k_z x_0^2}\right)^3\right)\right] \times \exp\left[a\frac{x}{x_0} - \frac{a}{2}\left(\frac{z}{k_z x_0^2}\right)^2\right].$$



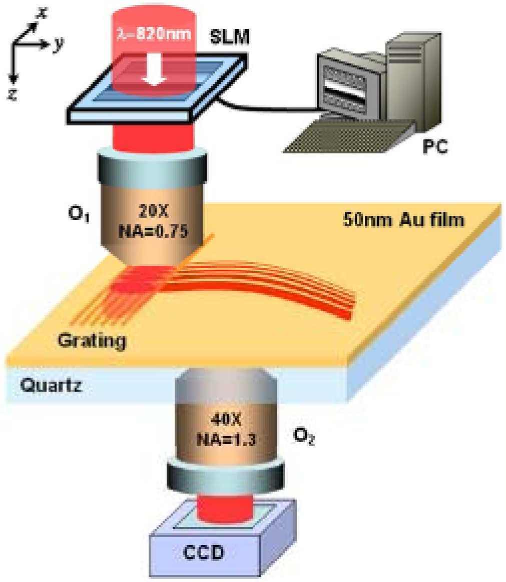
Only理论预测

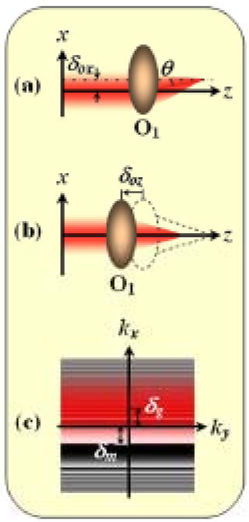
Nanjing University

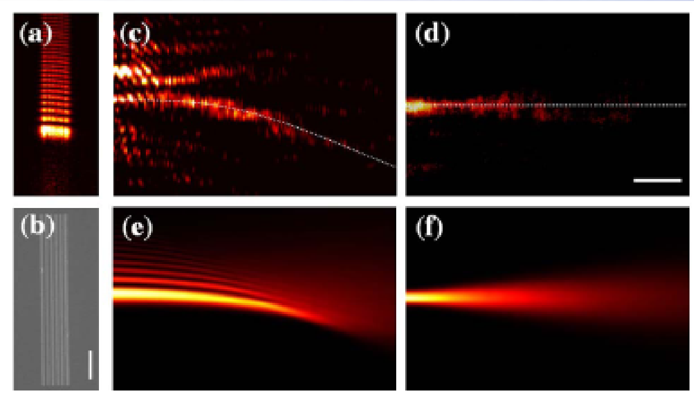
August 15, 2011 / Vol. 36, No. 16 / OPTICS LETTERS

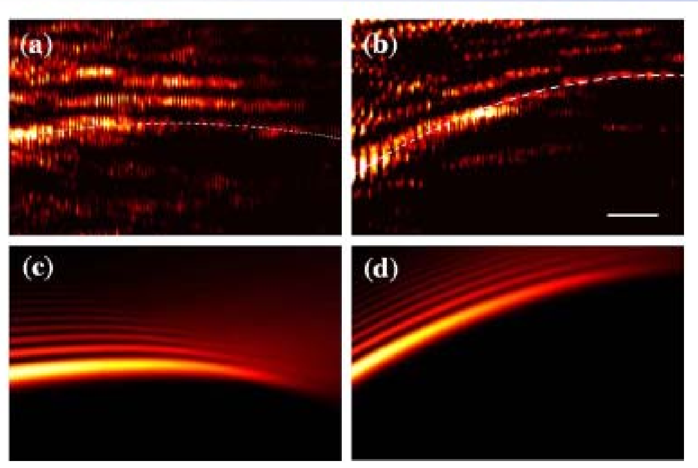
Plasmonic Airy beams with dynamically controlled trajectories

Peng Zhang,[1,2,†] Sheng Wang,[1,†] Yongmin Liu,[1,†] Xiaobo Yin,[1,3] Changgui Lu,[1] Zhigang Chen,[2] and Xiang Zhang[1,3,*]







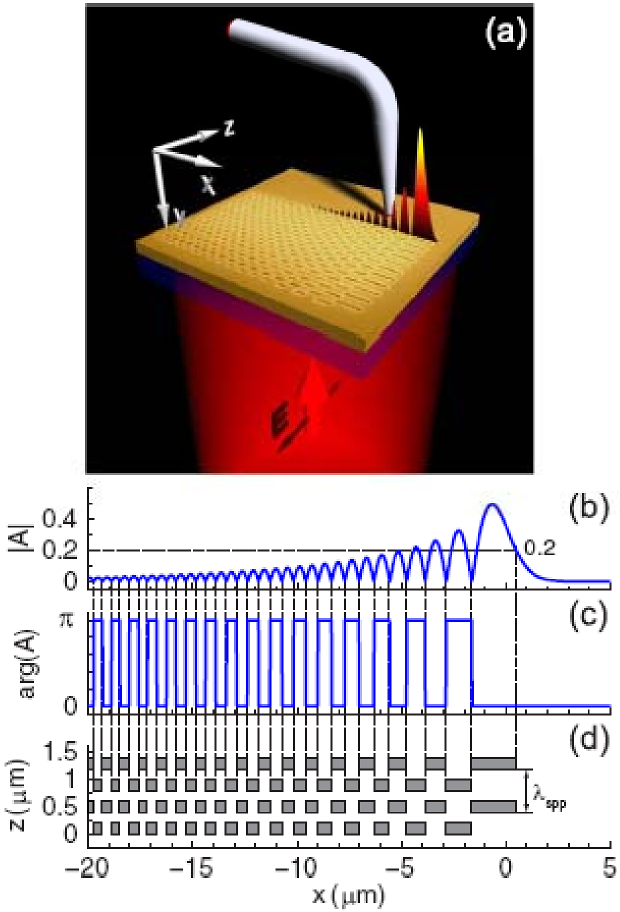


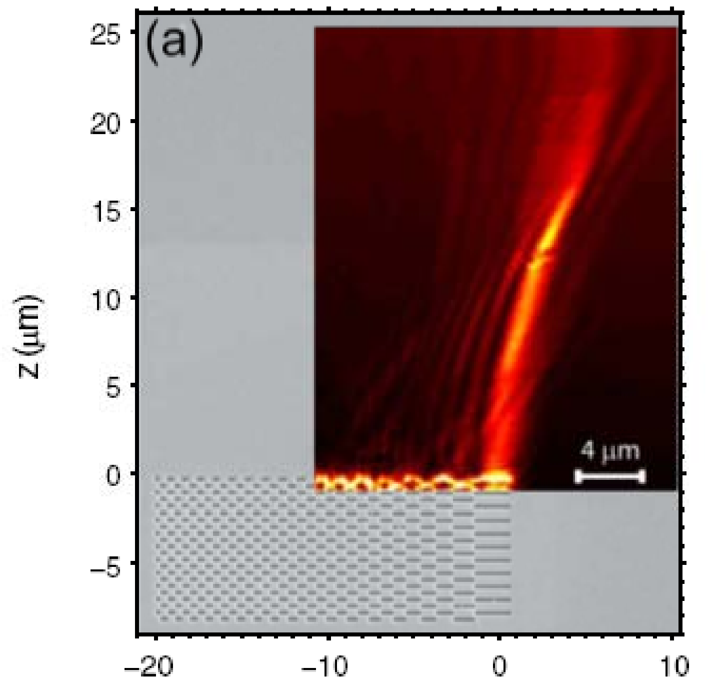
Coupling a well generated Airy beam to SPP

Selected for a Viewpoint in *Physics*
PRL **107,** 116802 (2011) PHYSICAL REVIEW LETTERS week ending 9 SEPTEMBER 2011

Generation and Near-Field Imaging of Airy Surface Plasmons

Alexander Minovich,[1] Angela E. Klein,[2] Norik Janunts,[2] Thomas Pertsch,[2] Dragomir N. Neshev,[1] and Yuri S. Kivshar[1]





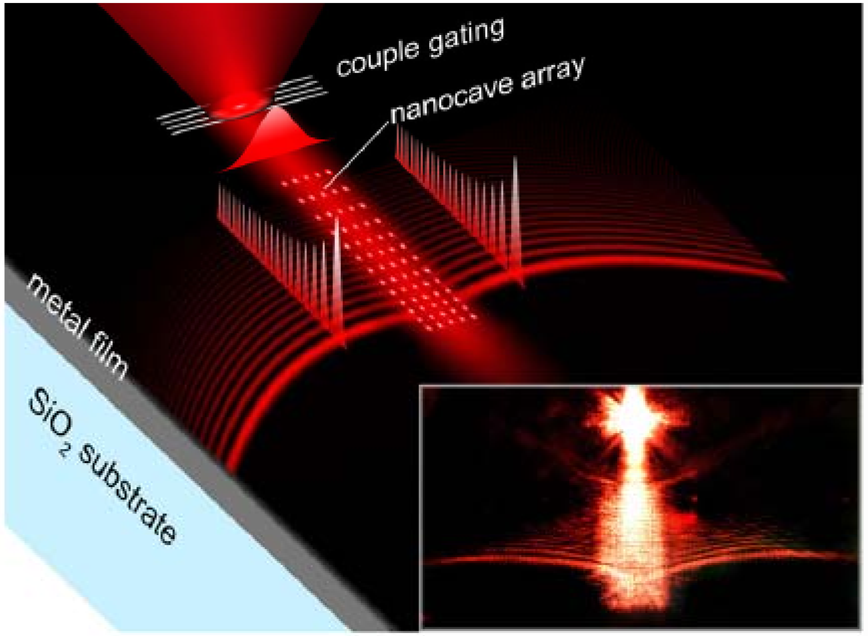
Using well designed nano-grating to couple Airy plasmon (coupling process)

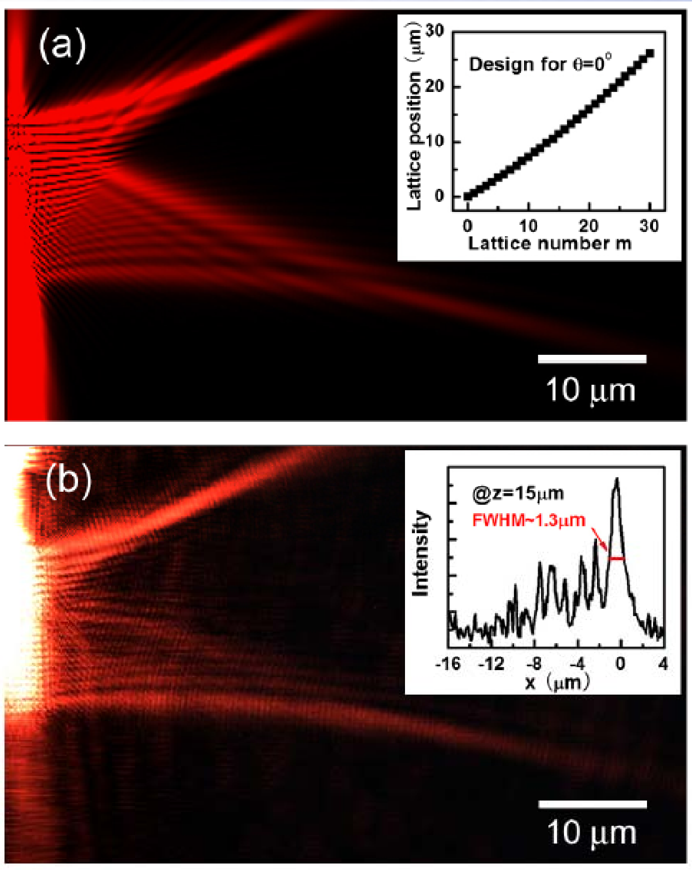
Nanjing University

PRL **107,** 126804 (2011) PHYSICAL REVIEW LETTERS week ending 16 SEPTEMBER 2011

Plasmonic Airy Beam Generated by In-Plane Diffraction

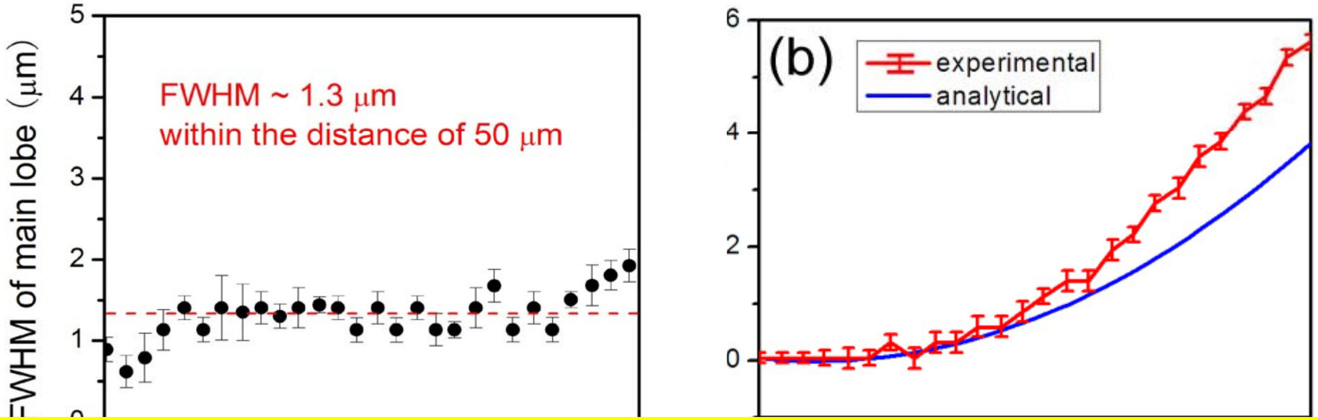
L. Li, T. Li,* S. M. Wang, C. Zhang, and S. N. Zhu



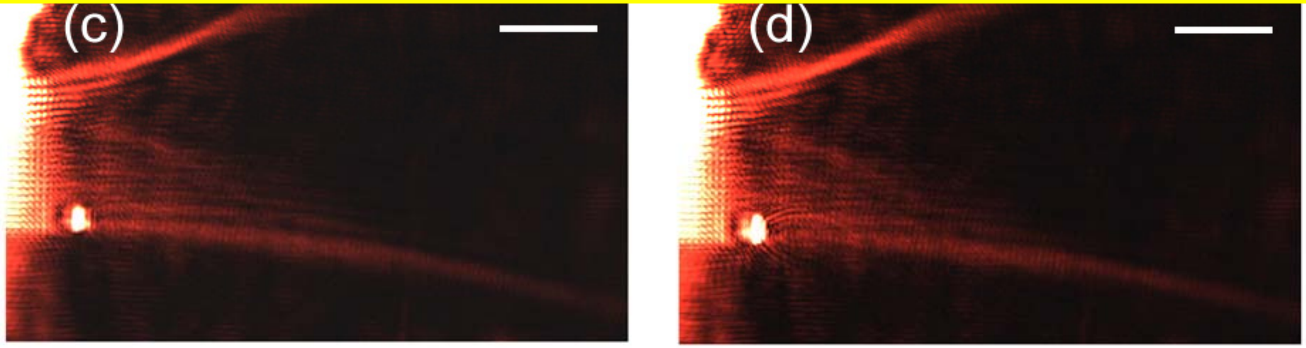


Generating Airy plasmon totally on planar dimension.
(independent to coupling process)

Non-spreading beam **Parabolic trajectory**

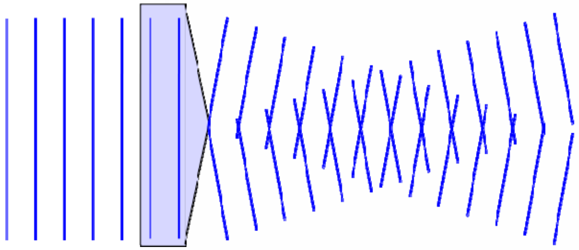


Q4：是否存在straight nondiffracting SPP?

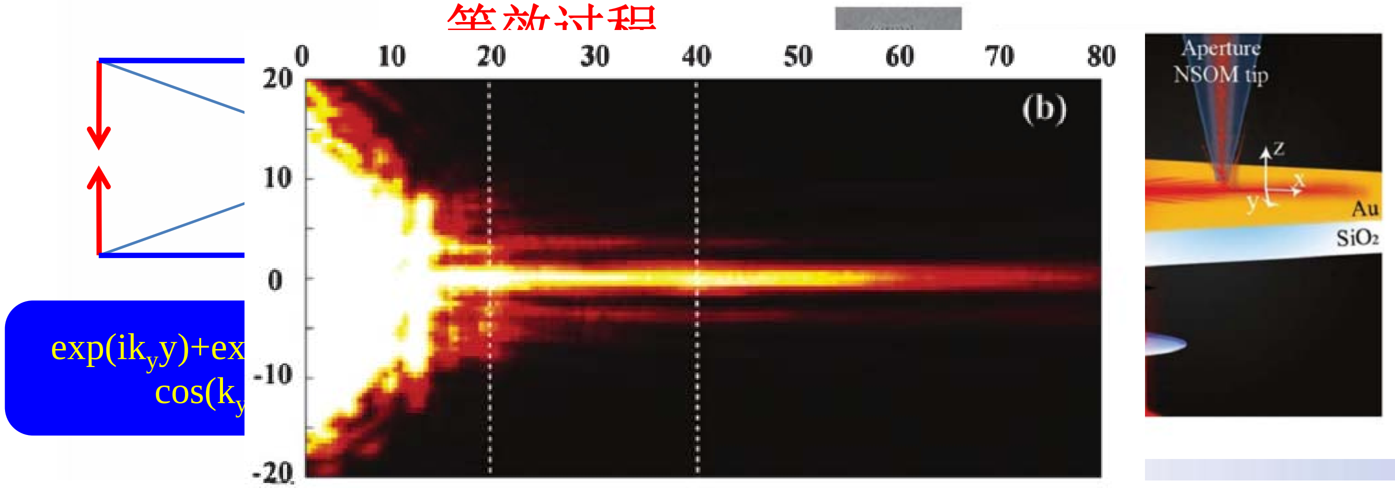


Self-healing property

Cosine-Gauss plasmon



Bessel beam是柱坐标下平面波的叠加
而SPP仅仅存在金属表面2D体系，不可能适用柱坐标！

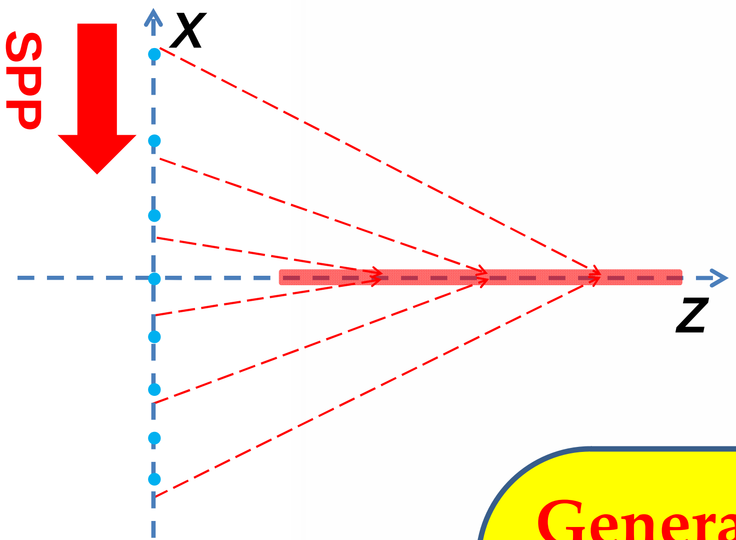


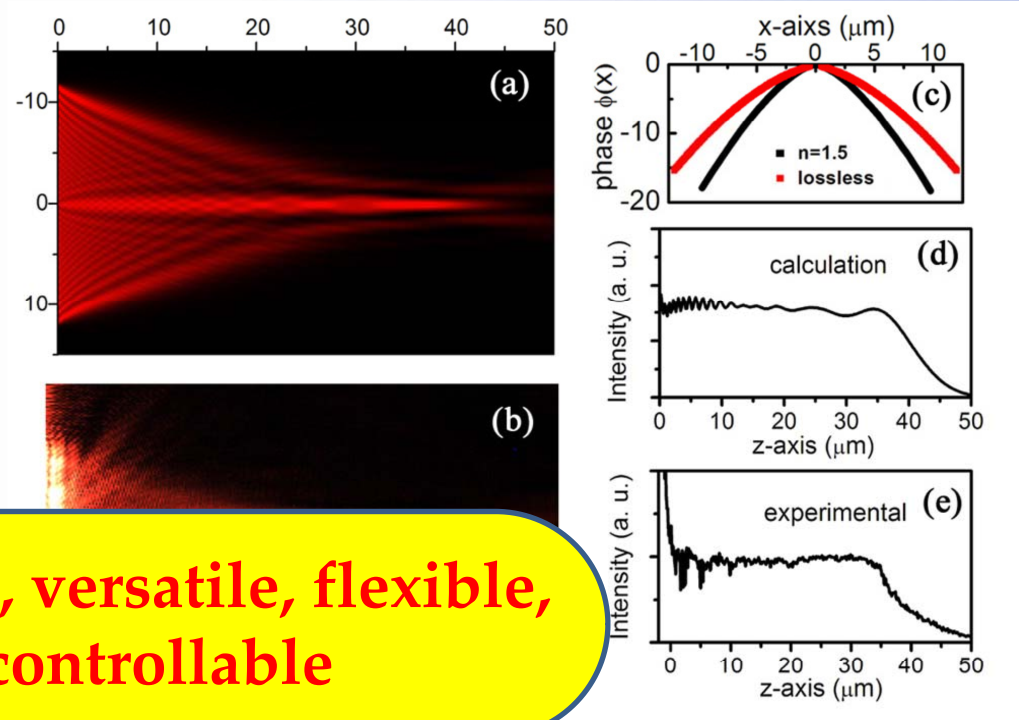
PRL **109,** 093904 (2012) PHYSICAL REVIEW LETTERS week ending 31 AUGUST 2012

Cosine-Gauss Plasmon Beam: A Localized Long-Range Nondiffracting Surface Wave

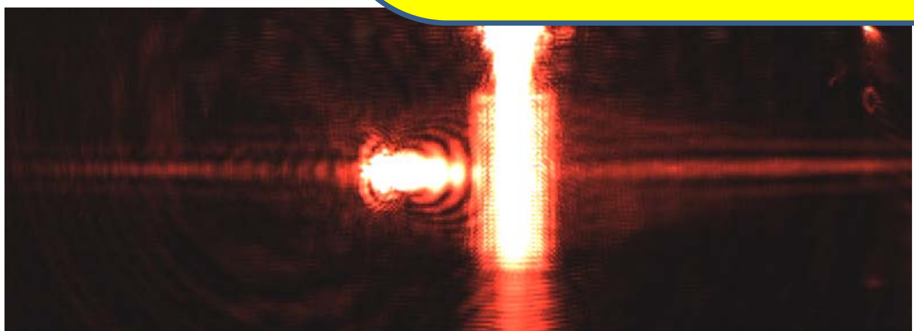
Jiao Lin,[1,2] Jean Dellinger,[3] Patrice Genevet,[1,4] Benoit Cluzel,[3] Frederique de Fornel,[3] and Federico Capasso[1,*]

Our strategy- diffraction process





General, versatile, flexible, controllable



"lossless" SPP beam

Self-healing property

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PRL 112, 023903 (2014) PHYSICAL REVIEW LETTERS week ending 17 JANUARY 2014

Arbitrary Bending Plasmonic Light Waves

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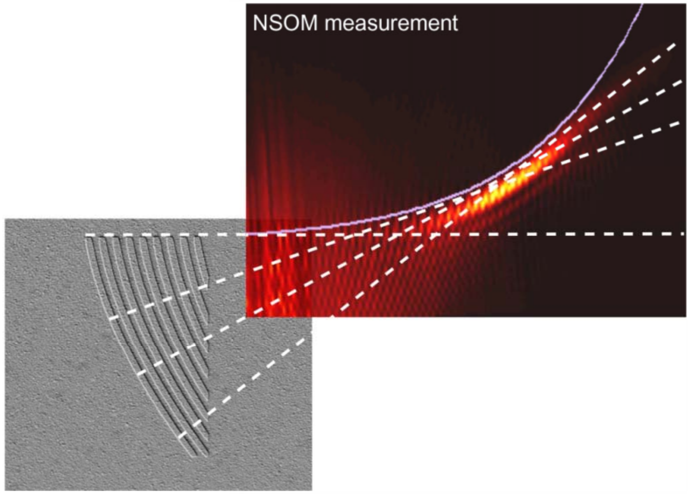
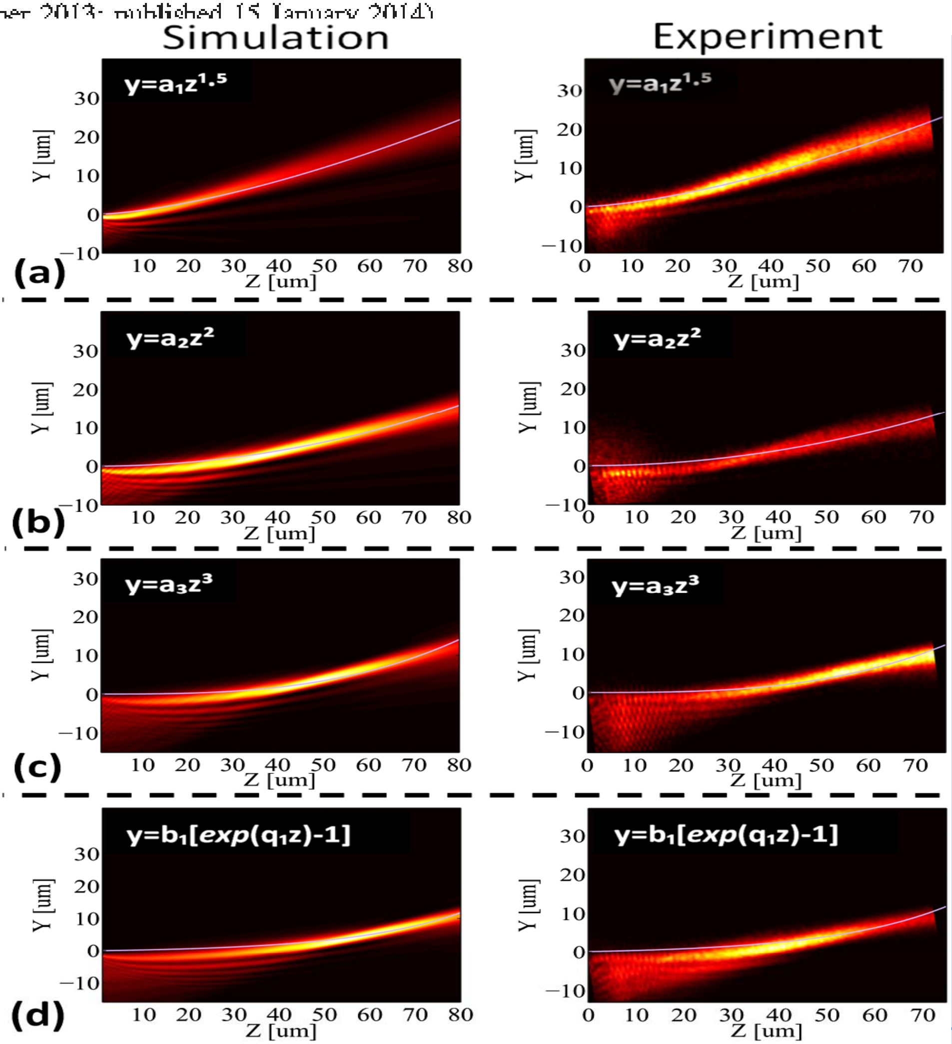
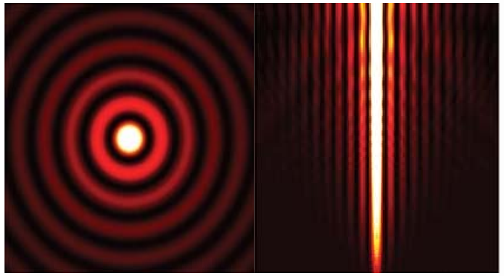
(Received 24 October 2013; published 15 January 2014)

How about nonparaxial SPP?

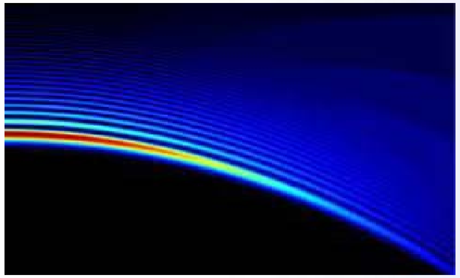
Caustic方法获得任意曲线传播



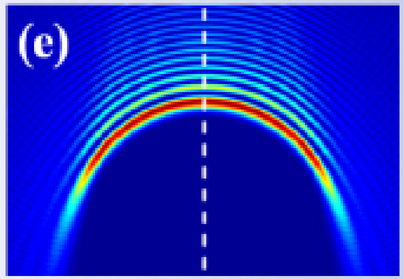




Bessel beam

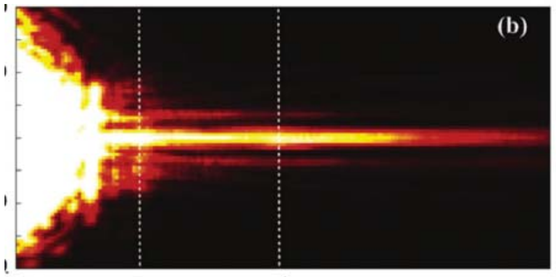


Airy Beam

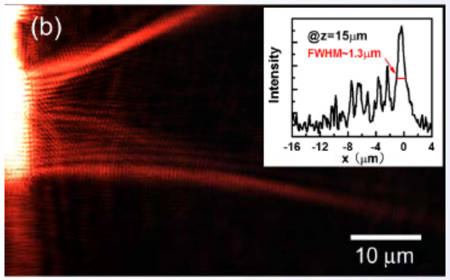


Mathieu Beam

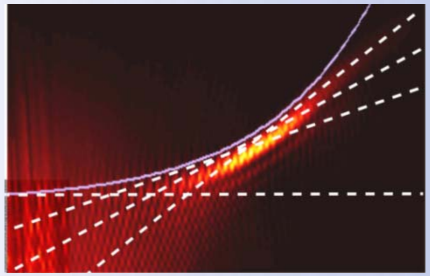
Nondiffracting beams	Bessel beam	Airy beam	Nonparaxial beam
Free space	1987	2007	2012
SPP	2012	2011	2014



Cosine-Gauss Plasmon



Airy Plasmon



Arbitrary bending SPP

Any more?

Use your imagination, also depends on your solid foundation!

需要ppt请登录
http://dsl.nju.edu.cn/litao/res/talk/

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References:

- Phys. Rev. Lett. 58, 1499 (1987)
- Phys. Rev. Lett. 99, 213901 (2007)
- Phys. Rev. Lett. 107, 116802 (2011)
- Phys. Rev. Lett. 107, 126804 (2011)
- Phys. Rev. Lett. 106, 213903 (2011)
- Phys. Rev. Lett. 106, 213902 (2011)
- Phys. Rev. Lett. 108, 113903 (2012)
- Phys. Rev. Lett. 108, 163901 (2012)
- Phys. Rev. Lett. 109, 093904 (2012)
- Phys. Rev. Lett. 109, 193901 (2012)
- Phys. Rev. Lett. 109, 203902 (2012)
- Phys. Rev. Lett. 109, 203903 (2012)
- Phys. Rev. Lett. 110, 046807 (2013)
- Phys. Rev. Lett. 112, 023903 (2014)
- Am. J. Phys. 47, 264 (1979)
- Opt. Lett. 34, 2634 (2009)
- Opt. Lett. 34, 3430 (2009)
- Opt. Express 18, 8440 (2010)
- Opt. Lett. 35, 2082 (2010)
- Opt. Lett. 36, 1164 (2010)
- Opt. Lett. 36, 3191 (2011)
- Opt. Express 19, 16455 (2011)
- Opt. Lett. 37, 2820 (2012)
- Opt. Lett. 37, 3402 (2012)
- Opt. Lett. 37, 5091 (2012)
- Opt. Lett. 38, 1733 (2013)
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